ameter also has an influence on frequency stability of Faraday rotation (Fig. 5). The combining of these two effects proved to give excellent results experimentally. The main advantage of this method lies in the possibility of using easily accessible and low-loss polystyrene instead of high permittivity dielectrics, and in the fact that the ferrite length has no effect on the broad-band performance. Fig. 6 shows the experimental results for Ferroxcube B5, obtained with a 3-inch-long sample of optimum diameter.

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Equivalence of 0 and −1 Space Harmonics in Helical Antenna Operation*

In considering the propagation of electromagnetic waves along helical conductors using the Tape Helix approximation, it is well known[1] that the solution contains an infinite number of space harmonics. The phase constants of these harmonics are related by

$$\beta_m = \beta_0 + \frac{2\pi m}{p},$$

where β_0 is the phase constant of the fundamental, p is the helical pitch and m is any integer including zero. It has been shown by Watkins[2] that as far as axial propagation is concerned, it is the −1 space harmonic which is responsible for the operation of the helical antenna. If, however, propagation along the conductor is considered, then the correct space harmonic to be considered is the fundamental as used originally by Sensiper.[3] It is easy to show that both approaches lead to identical results, the proof being as follows.

Let the phase shift between adjacent turns of the helix be denoted by θ with the subscript 0 or −1; depending on whether the fundamental or the −1 space harmonic is being considered. Then

$$\theta_0 = \frac{L}{\lambda_0}\cdot 2\pi$$

where L is the length of 1 helical turn and λ_0 is the fundamental wavelength. Denoting the axial velocity of the fundamental by v_0, the conductor phase velocity for the fundamental is $v_0/\sin\psi$, where ψ is the helical pitch angle, so that

$$\theta_0 = \frac{Lf}{\left(\dfrac{v_0}{\sin\psi}\right)}\cdot 2\pi$$

$$= \frac{2\pi pf}{v_0}.$$

Similarly,

$$\theta_{-1} = \frac{2\pi pf}{v_{-1}},$$

where v_{-1} is the axial phase velocity of the −1 space harmonic. This is related to the fundamental axial phase velocity v_0 by

$$\frac{v_{-1}}{v_0} = \frac{\beta_0 a}{\beta_0 a - \cot\psi}$$

so that θ_{-1} eventually simplifies to

$$\theta_{-1} = \frac{2\pi pf}{v_0} - 2\pi,$$

which is identical with the expression for θ_0 except for a difference of 2π which is not significant.

Therefore, it is equally valid to consider either the fundamental or the −1 space harmonic, the first relating to propagation along the conductor, and the second to propagation axially.

As these phase velocities apply to an infinite helix, it is not possible to use them directly for the finite antenna, since it has been found by Kraus[4] that the phase velocity is also a function of the length of the antenna. Nevertheless, it is known[5] that the solution for the infinite case may be used as a means of estimating the bandwidth of the antenna for any pitch angle ψ, and it is now shown that both axial and conductor propagation give identical results.

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[1] S. Sensiper, "EM wave propagation on helical structures," PROC. IRE, vol. 43, pp. 149–161; February, 1955.
[2] D. A. Watkins, "Topics in Electromagnetic Theory," John Wiley and Sons, Inc., New York, N. Y.; 1958.
[3] S. Sensiper, "E.M. wave propagation on Helical Conductors," M.I.T. Res. Lab. Tech. Rept. No. 194; 1951.
[4] J. D. Kraus, "Antennas," Mc-Graw-Hill Book Co., Inc., New York, N. Y.; 1950.
[5] T. S. M. Maclean and R. G. Kouyoumjian, "Bandwidth of the Uniform Helical Antenna," presented at URSI Symposium on Electromagnetic Theory, University of Toronto, Toronto, Can.; June, 1959.

Application of Perturbation Theory to the Calculation of ω-β Characteristics for Periodic Structures*

The effect of small periodic changes in the physical dimensions of closed periodic structures can be investigated using the perturbation theory developed by Müller[1] and later by Slater.[2] From this theory the fractional change in the natural frequency, ω, of a resonant cavity caused by the introduction into the cavity of a small conducting object of volume, τ, is given by

$$\delta\omega/\omega = \frac{1}{2}\,\frac{\int_\tau (\mu_0 H^2 - \epsilon_0 E^2)dV}{\int_v \epsilon_0 E^2 dV}. \qquad (1)$$

The integration in the numerator extends only over the volume of the perturbing object, whereas that in the denominator extends over the entire volume of the cavity, and E and H are the amplitudes of the electric and magnetic fields.

A commonly used technique for determining the ω-β characteristic for a closed periodic structure consists of constructing a resonator from an appropriately chosen length of the structure and determining the natural frequencies of the resonator which correspond to the field configurations of interest.[3] If the fields within the unperturbed structure are known, (1) may be used to compute the effect of small changes in the physical dimensions on these natural frequencies. This technique has been used by Vanhuyse[4] in the construction of a linear accelerator using a disk-loaded circular waveguide.

If the perturbations are periodic and if the period of the perturbation is an integral multiple of the fundamental period of the unperturbed structure, the resonant cavity technique may be used to determine the ω-β characteristic for the perturbed structure. For this case (1) may be used to relate the ω-β characteristic for the perturbed structure to that for the unperturbed structure.

As an illustration, let the initial unperturbed structure be a uniform disk-loaded circular waveguide of radius b, and let the perturbed structure comprise cavities alternately of radius b_- and b_+ as shown in Fig. 1.

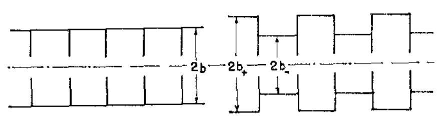

Fig. 1—Uniform and perturbed disk-loaded circular waveguides.

If the average volume per cell is unchanged by the perturbation and if $b_+ - b \ll b$, it is found that the ω-β characteristic for the perturbed structure coincides with that for the uniform structure except when the phase shift per section in the unperturbed structure is $\pi/2$. For this situation (which corresponds to a π phase shift per section in the perturbed structure), two frequencies are found, indicating the presence of a stop band. The width of the stop band is given by the difference between these two frequencies.

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[1] J. Muller, "Untersuchung über elektromagnetische hohlraume." *Hochfrequenz. und Elektroak.*, vol. 54, p. 157; November, 1939.
[2] J. C. Slater, "Microwave Electronics," D. Van Nostrand Co., Inc., Princeton, N. J., p. 80; 1950.
[3] B. Epsztein and G. Mourier, "Définition, mesure et caractères des vitesses de phase dans les systèmes á structure périodique" *Ann. Radioélectricité*, vol. 10, p. 64; January, 1955.
[4] V. J. Vanhuyse, "On the proper frequencies of terminated corrugated waveguides with slightly different diameters," *Physica*, vol. 21, p. 603; July, 1955.

Although calculations of this type do not predict the proper behavior for the ω-β characteristic near stopbands resulting from a periodic perturbation, they do predict the occurrence and width of such stop bands.

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Ice as a Bending Medium for Waveguide and Tubing*

Bending waveguide and metal tubing is very often a difficult and time-consuming task. Low melting temperature alloys are at times difficult to remove from waveguide and tubing. The piece to be bent may be filled with water which is then frozen by dry ice, liquid nitrogen, or by a deep freeze. In some applications where the piece to be bent is integral with a larger system, a block of dry ice may be held against it to freeze only the portion of water around the section to be bent. The use of these low temperatures causes not only the water to freeze into quite small crystals (which act like a sand packing), but also prevents the ice from melting because of the pressure of bending.

Several tests were performed on thin walled aluminum tubing and P-band brass waveguide. It was found that in comparison to low melting alloys the bends were identical within the statistical variation of samples. The time required for the operation was considerably shorter.

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On Higher-Order Hybrid Modes of Dielectric Cylinders*

In the course of investigations into the properties of various surface wave structures,[1] it became necessary to investigate hybrid modes on dielectric cylinders for modes of order n, where $n>1$. The case $n=1$ has received extensive treatment in the literature [1]–[6].

The radial dependence of the axial fields is as $J_n[p(\rho/a)]$ inside the dielectric cylinder and $K_n[q(\rho/a)]$ outside, where ρ is the radial cylindrical coordinate, a is the radius of the cylinder, p and q are radial eigenvalues, and n is the rank of the mode.

The requirement of continuity of the fields at the boundary leads, in the usual manner, to the characteristic equation involving Bessel functions and their derivatives. This was first given by Schelkunoff [4]. The derivatives of Bessel functions may be eliminated from this equation by the use of identities such as given by Watson [8], to yield the simple form

$$(J^+ + K^+)(\epsilon J^- - K^-) + (J^- - K^-)(\epsilon J^+ + K^+) = 0, \quad (1)$$

where

$$J^- = \frac{J_{n-1}(p)}{pJ_n(p)}, \quad J^+ = \frac{J_{n+1}(p)}{pJ_n(p)};$$
$$K^- = \frac{K_{n-1}(q)}{qK_n(q)}, \quad K^+ = \frac{K_{n+1}(q)}{qK_n(q)};$$

and ϵ is permittivity of dielectric cylinder relative to surrounding medium.

The cutoff values of the parameter p are of great interest; they may be obtained by letting $q\to 0$ in the characteristic equation. To keep the terms finite requires that the equation be multiplied by an appropriate power of q before the limit is taken. If it is assumed that J^- is finite at cutoff, it is sufficient to multiply the equation by q^2 to obtain a solution for the cutoff values of p; this was given by Schelkunoff [4]. However, if this assumption is not made, an additional solution may be determined. This will be outlined below.

Multiplying the characteristic equation by $[qpJ_n(p)]^2$ gives

$$(q^2J_{n+1} + q^2K^+pJ_n)(\epsilon J_{n-1} - pJ_nK^-) + (J_{n-1} - pJ_nK^-)(\epsilon q^2J_{n+1} + q^2K^+pJ_n) = 0. \quad (2)$$

Taking the limit as $q\to 0$ and noting that

$$K^- \to \frac{1}{2(n-1)}$$

and $q^2K^+\to 2n$ one obtains

$$2n\, pJ_n\left((\epsilon+1)J_{n-1} - \frac{pJ_n}{n-1}\right) = 0. \quad (3)$$

The solutions are, for $n>1$,

$$\frac{J_{n-1}(p)}{pJ_n(p)} = J^- = \frac{1}{(n-1)(\epsilon+1)}, \quad (4)$$
$$J_n(p) = 0, \quad p \neq 0. \quad (5)$$

Eq. 4 is given by Schelkunoff [4]. The very significant exclusion of the $p=0$ solution of (5) as a cutoff condition is based on the fact that for $q\to 0$ and $p\to 0$, (1) becomes, since

$$J^- \to \frac{2n}{p^2}, \quad J^+ \to \frac{1}{2(n+1)},$$

$$\left(\frac{1}{2(n+1)}+\frac{2n}{q^2}\right)\left(\frac{2n\epsilon}{p^2}-\frac{1}{2(n-1)}\right) + \left(\frac{2n}{p^2}-\frac{1}{2(n-1)}\right)\left(\frac{\epsilon}{2(n+1)}+\frac{2n}{q^2}\right) = 0. \quad (6)$$

When the finite terms are neglected in comparison with the infinite terms, it is seen that this is not satisfied at $q=0$, $p=0$ for any $n>1$. However, the $p=q=0$ solution, *i.e.*, the condition for "no cutoff," is valid for $n=1$ [1].

The asymptotes for the p-q curves are of interest. For $q\to\infty$ the characteristic equation becomes simply $2\epsilon J^-J^+=0$, with solutions at $J_{n-1}(p)=0$ and $J_{n+1}(p)=0$. It will be seen that the first of these is associated with the modes satisfying the first or Schelkunoff cutoff condition, the second with the alternate cutoff condition given here in (5).

Because of the oscillatory character of $J_n(p)$, the characteristic equation is satisfied by an infinite set of values of p for any given q, in particular also for $q=0$. These sets of p's span an infinite set of modes which may propagate along the dielectric rod. It is now seen that the existence of the alternate cutoff condition indicates the existence of an infinite set of modes that interlace the modes that satisfy the cutoff condition of (4). This and other salient characteristics of the doubly infinite set of modes are presented qualitatively in Fig. 1, with the $n=1$ case treated by Beam [1] included for comparison in Fig. 2. The curve shapes are based upon the detailed numerical solution of (2) obtained with an IBM 650 computer for $n=2$, 6 for a wide range of ϵ.

The significance of Fig. 1 may be summarized as follows.

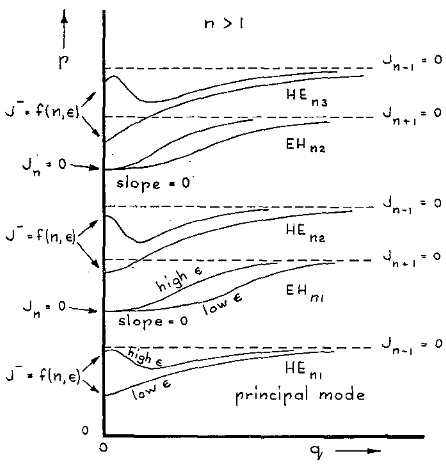

Fig. 1—Loci of solutions of the characteristic equation (1) for $n>1$.

$$f(n, \epsilon) = \frac{1}{(n-1)(\epsilon+1)}.$$

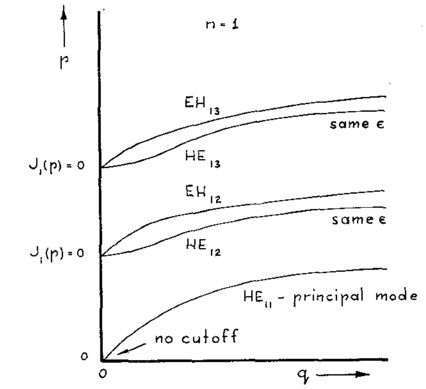

Fig. 2—Curves of p and q for $n=1$.

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[1] Report in preparation.